

Considerations On The Study Of Some Algebraic Equations In General Secondary Schools

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Abstract

This article discusses the algebraic equations $(x-a)(x-b)(x-c)(x-d)=A$, $(x-a)(x-b)(x-c)(x-d)=Ax^2$, as well as various O-symmetric, cosymmetric and inverse equations, and their solutions, using a sufficient number of examples.

Keywords: Algebraic equation, transcendental equation, symmetric equation, inverse equation, quadratic equation, cosymmetric equation.

It is known that today equations are taught from the first grade of general secondary schools. One of the main reasons for this is that equations are mathematical models of many problems encountered in practice. This requires special attention to the study of equations in general secondary schools and academic lyceums. The main part of the equations studied in general secondary schools is occupied by algebraic equations. An algebraic equation is generally written as follows:

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + ax + a = 0 \quad (1)$$

Here $a_0 \neq 0$, a_i ($i=1,2,3,\dots$) are the coefficients of the algebraic equation.

In an algebraic equation, when $n=1$, $n=2$, linear and quadratic equations are formed, and there are general methods for solving them. When $n=3$ and $n=4$, there are some methods for solving. When $n \geq 3$, there are no general methods for solving the equation. In such cases, when solving equations, an attempt is made to factor the left side or to solve it approximately. ≥ 3 When n , there are special forms of algebraic equations.

Algebraic equations can be divided into integer, fractional, rational and irrational equations. (1) Knowing the root of the

equation $x=\alpha$, we can reduce the degree of the given equation by one using Bezou's theorem.

We have already noted above that there are no general methods for solving algebraic equations of the form (1) ($n \geq 3$). When solving them, one mainly tries to factor them. The following example can be given.

Example 1. Solve the equation $x^3 + 2x^2 + 3x + 6 = 0$.

Solution: Factor the left side of the equation. $(x^3 + 2x^2) + (3x + 6) = 0$, $x^2(x + 2) + 3(x + 2) = 0$, $(x + 2)(x^2 + 3) = 0$. From this we write $x + 2 = 0$ and $x^2 + 3 = 0$. We know the methods for solving such equations.

Example 2. Solve the equation $x^4 + x^3 + 3x^2 + 2x + 2 = 0$.

Solution: It is not possible to factor the left side of this equation by grouping it all at once. Therefore, here we first write $3x^2$ as $2x^2 + x^2$, and then factor it all together.

$(x^4 + x^3 + x^2) + (2x^2 + 2x + 2) = 0$, $x^2(x^2 + x + 1) + 2(x^2 + x + 1) = 0$, $(x^2 + x + 1)(x^2 + 2) = 0$. From this

We form the equations $x^2 + x + 1 = 0$ and $x^2 + 2 = 0$. These equations do not have real roots.

Example 3. Solve the equation $x^3 + 4x^{2-24} = 0$.

Solution: This equation cannot be solved by the above methods. We will try to find all the roots of this equation. If it has all the roots,

they can be the divisors of the free term, namely $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$. We determine by the method of selection that $x=2$ is a root of this equation. In this case, the left side of the given equation is divided by $x-2$. After performing the division, we write the given equation in the form $(x-2)(x^2+6x+12)=0$. From this, we write $x-2=0$ and $x^2+6x+12=0$. We know the method of solving these.

Example 4. Solve the equation $(x^2+x+4)^2+8x(x^2+x+4)+15x^2=0$.

Solution: This equation cannot be solved by the above methods. To solve it, we substitute $y=x^2+x+4$. In this case, the equation $y^2+8xy+15x^2=0$ is obtained. We solve the resulting equation as a quadratic equation with respect to y .

$$y_{1,2} = -4x \pm \sqrt{16x^2 - 15x^2} = -4x \pm xy_1 = -3x, y_2 = -5x$$

So, we have the equations $x^2+x+4 = -3x$ and $x^2+x+4 = -5x$. We know the methods for solving them.

Example 5. Solve the equation $21x^3 + x^2 - 5x - 1 = 0$.

Solution: The given equation is an integral equation with the free term equal to -1 . This equation cannot be solved by any of the above methods. To solve this equation, we divide each term by the highest power of x . $21x^3 + x^2 - 5x - 1 = 0$, $21 + \frac{1}{x} - \frac{5}{x^2} - \frac{1}{x^3} = 0$ $\frac{1}{x} = y$.

Then

$$21+y-5y^2-y^3=0, \text{ that is, } y^3+5y^2-y-21=0, \text{ is formed.}$$

By the method of selection, we find that $y=-3$ is one of the roots of the given equation. Indeed, $(-3)^3+5(-3)^2-(-3)-21 = -27+45+3-21=0, 0=0$. Now, dividing both sides of the last equation by $x+3$, we get $(y+3)(y^2+2y-7)=0$, and from this we have the equations $y+3=0$ and $y^2+2y-7=0$. Taking into account the notation, we find the values of x .

Some algebraic equations can be written in the form $(x-a)(xb)(xc)(xd)=A$. Here a, b, c, d, A are the given numbers. If in this equation $a+b=c+d=k$, then the given

equation $y=x^2-kx$ can be solved by reducing it to a quadratic equation.

Example 6. Solve the equation $(x+1)(x+2)(x+3)(x+4)=24$.

Solution: Here $a = -1, b = -2, c = -3, d = -4$ and $A=24$, for which

$-1+(-4) = -2+(-3) = -5$ is valid. Therefore, we find the products $(x+1)(x+4)$ and $(x+2)(x+3)$ and get the equation $(x^2+5x+4)(x^2+5x+6)=24$. Now, if we take $y = x^2+5x$, we get the equation $(y+4)(y+6)=24$ or $y^2+10y=0$.

This gives $y_1=0$ and $y_2=-10$. Now, returning to the old variable, we obtain the equations $x^2+5x=0$ and $x^2+5x=-10$. We know the methods for solving these equations.

Some algebraic equations are also given in the form $(x-a)(xb)(xc)(xd)=Ax^2$. Here a, b, c, d, A are the given numbers. If in this equation $a+d=bc$, then the given equation $\frac{m}{x}$ is reduced to a quadratic equation using substitution $y=x+$.

7. Solve the equation $(x-2)(x-3)(x-4)(x-6)=30x^2$.

Solution: Here $a = 2, b = 3, c = 4, d = 6$ and $A=30$. Here $2 \cdot 6 = 3 \cdot 4 = 12$. Now $(x-3)(x-4)$ and $(x-2)(x-6)$ and form the following equation.

$$(x^2-7x+12)(x^2-8x+12)=30x^2$$

each bracket on the left side of this equation by x and the right side by x^2 . As a result, the following equation is formed.

$$(x + \frac{12}{x} - 7)(x + \frac{12}{x} - 8) = 30$$

by substituting $y = x + \frac{12}{x}$, we get the equation $(y-7)(y-8)=30$ or $y^2-15y+26=0$, and from this we get $y_1=2, y_2=13$. Returning to the old variable, we get the equations $x + \frac{12}{x} = 2$ and $x + \frac{12}{x} = 13$, and from these we get the quadratic equations $x^2-2x+12=0$ and $x^2-13x+12=0$. We know the methods for solving these equations.

Sometimes there are equations of the form $ax^4+bx^3+cx^2+bx+a=0, a \neq 0$. Observing this equation, we see that the coefficients of the leading coefficient and the free term, as well as the terms equidistant from them, are

equal to each other. Usually such equations are called fourth-degree symmetric equations. Such equations $\frac{1}{x}$ are reduced to a quadratic equation using the substitution $y=x+\frac{1}{x}$.

Example 8. Solve the equation $x^4+5x^3+2x^2+5x+1=0$.

Solution: This equation is a fourth-degree symmetric equation. To solve it, we divide by x^2 ($x \neq 0$) and group the terms as follows.

$$x^2+5x+2+\frac{5}{x}+\frac{1}{x^2}=0, (x^2+\frac{1}{x^2})+5(x+\frac{1}{x})+2=0$$

$y=x+\frac{1}{x}$, then $x^2+\frac{1}{x^2}=y^2-2$, and the last equation becomes $y^2-2+5y+2=0$ or $y^2+5y=0$.

From this we find $y_1=0$, $y_2=-5$. Now, if we return to the old variable $x+\frac{1}{x}=0$ and $x+\frac{1}{x}=-5$ or $x^2+1=0$ and $x^2+5x+1=0$, we come to the equations. We know the method for solving these equations.

$ax^3+bx^2+bx+a=0$, An equation of the form $a \neq 0$ is called a cubic symmetric equation. It is clear that the number $x=-1$ is one of the roots of this equation. Therefore, we get a quadratic equation whose solution is known by dividing the left side of this equation by $x+1$.

Example 9. Solve the equation $2x^3+7x^2+7x+2=0$.

Solution: The given equation is a cubic symmetric equation. $x=-1$ is a root of this equation. In fact, $2(-1)^3+7(-1)^2+7(-1)+2=-2+7-7+2=0$. We divide the left side of the equation by $x+1$ to obtain the division $2x^2+5x+2$. Therefore, we can write the given equation in the form $(x+1)(2x^2+5x+2)=0$. This equation is equivalent to the equations $x+1=0$ and $2x^2+5x+2=0$. We know the methods for solving these.

The equation $ax^5+bx^4+cx^3+cx^2+bx+a=0$, $a \neq 0$ is called a fifth-degree symmetric equation. One of the roots of this equation is $x=-1$. We divide the left side of the equation by $x+1$ to obtain a fourth-degree symmetric equation. We know the method for solving it.

Example 10. Solve the equation $x^5+2x^4+3x^3+3x^2+2x+1=0$.

Solution: It is clear that $x=-1$ is one of the roots of the equation. Dividing the left side of the equation by $x+1$, we obtain the equation $x^4+x^3+2x^2+x+1=0$. This is a fourth-degree symmetric equation, and we know how to solve it.

Sometimes we also encounter equations of the form $ax^4+bx^3+cx^2-bx+a=0$, $a \neq 0$. Such an equation is called a fourth-degree cosymmetric equation. This equation can be reduced to a quadratic equation using the substitution $y=x-\frac{1}{x}$.

Example 11. Solve the equation $6x^4+25x^3+12x^2-25x+6=0$.

We reduce both sides of the equation to x^2 ($x \neq 0$). $6x^2+25x+12-\frac{25}{x}+\frac{6}{x^2}=0$, $6(x^2+\frac{1}{x^2})+25(x-\frac{1}{x})+12=0$. If $y=x-\frac{1}{x}$ we take $y=x-\frac{1}{x}$, then $x^2+\frac{1}{x^2}=y^2+2$. Taking these into account, we get the quadratic equation $6(y^2+2)+25y+12=0$ or $6y^2+25y+24=0$. Solving it, we find $y_1=-\frac{8}{3}$, $y_2=-\frac{3}{2}$. Now, returning to the old variable, we get the following equations.

1) $x-\frac{1}{x}=-\frac{8}{3}$; 2) $x-\frac{1}{x}=-\frac{3}{2}$. We know the methods for solving this equation.

In practice, fifth- and third-degree cosymmetric equations of the form $ax^5+bx^4+cx^3-cx^2-bx-a=0$, $a \neq 0$ and $ax^3+bx^2-bx-a=0$, $a \neq 0$ are also encountered. The number $x=1$ is a root of each of these equations. Therefore, each of them Dividing by $x-1$ lowers the degree of the given equation by one.

Example 12. Solve the equation $x^5-4x^4+5x^3-5x^2+4x-1=0$.

Solution: The given equation is a cosymmetric equation of the fifth degree. It is clear that one of its roots is $x=1$. Indeed, $1^5-4 \cdot 1^4+5 \cdot 1^3-5 \cdot 1^2-4 \cdot 1-1=1-4+5-5+4-1=0$. Divide the given equation by $x-1$,

We form a symmetric equation of the fourth degree $x^4-3x^3+2x^2-3x+1=0$. We know the method for solving this equation.

Another one of the most common algebraic equations

$\alpha x^4 + \beta x^3 + \gamma x^2 + \lambda \beta x + \lambda^2 \alpha = 0$ is an equation. If we observe this equation, we see that the ratio of the coefficient of the first term to the free term, and the coefficient of the second term is equal to the square of the ratio of the coefficient of the last term to the coefficient of the previous term. Such equations are called fourth-degree inverse equations. When solving such equations, the degree is reduced by two using the substitution $y = x + \frac{\lambda}{x}$.

Example 13: Solve the equation $3x^4 - 2x^3 + 4x^2 - 4x + 12 = 0$.

This equation is a quadratic inverse equation. In this equation, we also have $\frac{3}{12} = (\frac{-2}{-4})^2$, $\frac{1}{4} = \frac{1}{4}$. So the given equation is a quadratic inverse equation. To solve it, we divide by x^2 ($x \neq 0$).

$3x^2 - 2x + 4 - \frac{4}{x} + \frac{12}{x^2} = 0$, $3(x^2 + \frac{4}{x^2}) - 2(x + \frac{2}{x}) + 4 = 0$.

We substitute $x + \frac{2}{x} = y$. Then $x^2 + \frac{4}{x^2} = y^2 - 4$.

Taking these into account, we get the following equation:

$3(y^2 - 4) - 2y + 4 = 0$, $3y^2 - 2y - 8 = 0$; $y_1 = 2$, $y_2 = -\frac{4}{3}$. Now, if we return to the old variable, the following equations are formed.

$x + \frac{2}{x} = 2$; $x + \frac{2}{x} = -\frac{4}{3}$. We know the methods for solving such equations.

The equation $\alpha x^6 + \beta x^5 + \gamma x^4 + \delta x^3 + \lambda \gamma x^2 + \lambda^2 \beta x + \lambda^3 \alpha = 0$, $\alpha \neq 0$ is a sixth-degree inverse equation. Such equations can also be solved by the above method. We observe the symmetric, cosymmetric, and inverse equations and see that the first two equations are cases of inverse equations with $\lambda = 1$ and $\lambda = -1$.

So in this article we will be referring to the square label

$(x - \alpha)(x - \beta)(x - \gamma) = A$, $(x - \alpha)(x - \beta)(x - \gamma) = Ax^2$.

In practice, there are equations that cannot be included in the above class of equations. Such equations are usually solved by approximate methods. In the

Vol 3. Issue 2 (2026)

process of studying such equations, students' creative thinking skills are further developed and serve as an important factor in their thorough mastery of transcendental equations, and as a result, mathematical education becomes effective. This can be considered a timely step towards implementing the Decree of the President of our Republic dated July 9, 2019 "On state support for the development of mathematical sciences and education, as well as measures to radically improve the activities of the V. I. Romanovsky Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan".

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